1 マクスウェル方程式

1.1 電場のガウスの法則

- 電束密度の発散は電荷密度.
- 電荷があれば, 電場が発散.
- $\boldsymbol{D}$ は電束密度, ρ は電荷密度である.

$$
\begin{aligned}
\int_S \boldsymbol{D} \cdot \boldsymbol{n} dS &= \int_V \rho dV \\
\int_V \nabla \cdot \boldsymbol{D} dV &= \int_V \rho dV \\
\int_V (\nabla \cdot \boldsymbol{D} - \rho) dV &= 0 \\
\nabla \cdot \boldsymbol{D} - \rho &= 0 \\
\nabla \cdot \boldsymbol{D} &= \rho \\
\mathrm{div}\ \boldsymbol{D} &= \rho \qquad (1)
\end{aligned}
$$

1.2 磁場のガウスの法則

- 磁束密度の発散は 0.
- 磁場は発散しない.
- $\boldsymbol{B}$ は磁束密度である.

$$
\begin{aligned}
\int_S \boldsymbol{B} \cdot \boldsymbol{n} dS &= 0 \\
\int_V \nabla \cdot \boldsymbol{B} dV &= 0 \\
\nabla \cdot \boldsymbol{B} &= 0 \\
\mathrm{div}\ \boldsymbol{B} &= 0 \qquad (2)
\end{aligned}
$$

1.3 ファラデー・マクスウェルの法則

- 電場の回転は, 磁束を妨げる方向に磁束密度の時間変化分となる.
- $\boldsymbol{E}$ は 電場, $\boldsymbol{B}$ は磁束密度である.

$$
\begin{aligned}
\oint_C \boldsymbol{E} \cdot d\boldsymbol{r} &= -\int_S \frac{\partial \boldsymbol{B}}{\partial t} \cdot \boldsymbol{n} dS \\
\int_S (\nabla \times \boldsymbol{E}) \cdot \boldsymbol{n} dS &= -\int_S \frac{\partial \boldsymbol{B}}{\partial t} \cdot \boldsymbol{n} dS \\
\int_S \left(\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} \right) \cdot \boldsymbol{n} dS &= 0 \\
\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} &= 0 \\
\nabla \times \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t} \\
\mathrm{rot}\ \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t} \qquad (3)
\end{aligned}
$$

1.4 アンペール・マクスウェルの法則

- 磁場の回転は, 電流密度 + 電束密度$\boldsymbol{D}$の時間変化分 となる.
- $\boldsymbol{H}$ は 磁場, $\boldsymbol{J}$ は電流密度, $\boldsymbol{D}$ は電束密度である.

$$\oint_C \boldsymbol{H} \cdot d\boldsymbol{r} = \underbrace{\int_S \boldsymbol{J} \cdot \boldsymbol{n} dS}_{I} + \int_S \frac{\partial \boldsymbol{D}}{\partial t} \cdot \boldsymbol{n} dS$$

$$\int_S (\nabla \times \boldsymbol{H}) \cdot \boldsymbol{n} dS = \int_S \boldsymbol{J} \cdot \boldsymbol{n} dS + \int_S \frac{\partial \boldsymbol{D}}{\partial t} \cdot \boldsymbol{n} dS$$

$$\int_S \left(\nabla \times \boldsymbol{H} - \boldsymbol{J} - \frac{\partial \boldsymbol{D}}{\partial t} \right) \cdot \boldsymbol{n} dS = 0$$

$$\nabla \times \boldsymbol{H} - \boldsymbol{J} - \frac{\partial \boldsymbol{D}}{\partial t} = 0$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

$$\mathrm{rot}\ \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \tag{4}$$

1.5 $\boldsymbol{D}$, $\boldsymbol{H}$, E, $\boldsymbol{M}$, $\boldsymbol{B}$

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \underbrace{\boldsymbol{P}}_{\text{分極ベクトル}} \tag{5}$$

$$\boldsymbol{H} = \frac{1}{\mu_0} \boldsymbol{B} - \underbrace{\boldsymbol{M}}_{\text{磁化ベクトル}} \tag{6}$$

$$\boldsymbol{M} = \underbrace{\chi_m}_{\text{磁化率}} \boldsymbol{H} \tag{7}$$

$$\boldsymbol{B} = \mu_0 \underbrace{\mu_r}_{1+\chi_m\ \text{(比透磁率)}} \boldsymbol{H} \tag{8}$$

$$E = \frac{V}{d} \tag{9}$$

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \tag{10}$$

$$\mathrm{grad}\ A = \nabla A = \left(\frac{\partial A}{\partial x}, \frac{\partial A}{\partial y}, \frac{\partial A}{\partial z} \right) = \boldsymbol{B} \tag{11}$$

$$\mathrm{div}\ \boldsymbol{A} = \nabla \cdot \boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = C \tag{12}$$

2 ローレンツ力

$$\boldsymbol{F} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B} \tag{13}$$

3 電荷保存則 (連続の式)

$$
\begin{aligned}
-\frac{d}{dt}\int_V \rho dV &= \int_S \boldsymbol{J}\cdot\boldsymbol{n}dS \\
\frac{\partial\rho}{\partial t} + \nabla\cdot\boldsymbol{J} &= 0
\end{aligned} \tag{14}
$$

4 ガウスの発散定理とストークスの定理

$$
\int_S \boldsymbol{D}\cdot\boldsymbol{n}dS = \int_V \nabla\cdot\boldsymbol{D}dV \tag{15}
$$

$$
\oint_C \boldsymbol{E}\cdot d\boldsymbol{r} = \int_S (\nabla\times\boldsymbol{E})\cdot\boldsymbol{n}dS \tag{16}
$$

5 電気素量

$$
e = 1.60217663\times 10^{-19}\mathrm{C} \tag{17}
$$

6 電磁場のエネルギー保存則

- エネルギー密度の時間変化と, 電磁場のエネルギー ($\boldsymbol{S}$)の発散 の和が 0, つまり一定に保たれる.

$$
\begin{aligned}
\nabla\times\boldsymbol{H} - \frac{\partial\boldsymbol{D}}{\partial t} &= \boldsymbol{J} \\
\boldsymbol{E}\cdot\left(\nabla\times\boldsymbol{H} - \frac{\partial\boldsymbol{D}}{\partial t}\right) &= \boldsymbol{E}\cdot\boldsymbol{J}
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
\nabla\times\boldsymbol{E} + \frac{\partial\boldsymbol{B}}{\partial t} &= 0 \\
\boldsymbol{H}\cdot\left(\nabla\times\boldsymbol{E} + \frac{\partial\boldsymbol{B}}{\partial t}\right) &= 0
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
\boldsymbol{H}\cdot\left(\nabla\times\boldsymbol{E} + \frac{\partial\boldsymbol{B}}{\partial t}\right) - \boldsymbol{E}\cdot\left(\nabla\times\boldsymbol{H} - \frac{\partial\boldsymbol{D}}{\partial t}\right) &= -\boldsymbol{E}\cdot\boldsymbol{J} \\
\boldsymbol{H}\cdot(\nabla\times\boldsymbol{E}) - \boldsymbol{E}\cdot(\nabla\times\boldsymbol{H}) + \boldsymbol{H}\cdot\frac{\partial\boldsymbol{B}}{\partial t} + \boldsymbol{E}\cdot\frac{\partial\boldsymbol{D}}{\partial t} &= -\boldsymbol{E}\cdot\boldsymbol{J}
\end{aligned} \tag{20}
$$

$$
\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H}) = \boldsymbol{H}\cdot(\nabla\times\boldsymbol{E}) - \boldsymbol{E}\cdot(\nabla\times\boldsymbol{H}) \tag{21}
$$

$$
\begin{aligned}
\frac{\partial\boldsymbol{E}^2}{\partial t} &= \frac{\partial\boldsymbol{E}\cdot\boldsymbol{E}}{\partial t} = \frac{\partial\boldsymbol{E}}{\partial t}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\frac{\partial\boldsymbol{E}}{\partial t} = 2\boldsymbol{E}\cdot\frac{\partial\boldsymbol{E}}{\partial t} \\
\frac{1}{2}\frac{\partial\boldsymbol{E}^2}{\partial t} &= \boldsymbol{E}\cdot\frac{\partial\boldsymbol{E}}{\partial t}
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
\boldsymbol{H}\cdot\frac{\partial\boldsymbol{B}}{\partial t} + \boldsymbol{E}\cdot\frac{\partial\boldsymbol{D}}{\partial t} &= \varepsilon_0\boldsymbol{E}\cdot\frac{\partial\boldsymbol{E}}{\partial t} + \mu_0\boldsymbol{H}\cdot\frac{\partial\boldsymbol{H}}{\partial t} = \frac{\varepsilon_0}{2}\frac{\partial\boldsymbol{E}^2}{\partial t} + \frac{\mu_0}{2}\frac{\partial\boldsymbol{H}^2}{\partial t} \\
&= \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon_0\boldsymbol{E}^2 + \frac{1}{2}\mu_0\boldsymbol{H}^2\right) = \frac{\partial}{\partial t}\left(\frac{1}{2}\boldsymbol{E}\cdot\boldsymbol{D} + \frac{1}{2}\boldsymbol{B}\cdot\boldsymbol{H}\right)
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) + \frac{\partial}{\partial t}\left(\frac{1}{2}\boldsymbol{E} \cdot \boldsymbol{D} + \frac{1}{2}\boldsymbol{B} \cdot \boldsymbol{H}\right) = -\boldsymbol{E} \cdot \boldsymbol{J} \\
\underbrace{\frac{\partial u}{\partial t}}_{\frac{\partial}{\partial t}\left(\frac{1}{2}\boldsymbol{E} \cdot \boldsymbol{D} + \frac{1}{2}\boldsymbol{B} \cdot \boldsymbol{H}\right)} + \nabla \cdot \underbrace{\boldsymbol{S}}_{\boldsymbol{E} \times \boldsymbol{H}} = \underbrace{0}_{-\boldsymbol{E} \cdot \boldsymbol{J}}
\end{aligned} \tag{24}
$$

7 境界

$$
\begin{aligned}
\boldsymbol{n} \times (\boldsymbol{H}_1 - \boldsymbol{H}_2) = 0 \\
\boldsymbol{n} \cdot (\boldsymbol{B}_1 - \boldsymbol{B}_2) = 0
\end{aligned} \tag{25}
$$

8 サイクロトロン

$$
\begin{aligned}
\frac{mv^2}{R} = qvB \\
\omega = \frac{v}{R} = \frac{qB}{m}
\end{aligned} \tag{26}
$$